

EXCELLENCE QUESTIONS I

Excellence questions may involve:

- Knowledge of focus/foci, eccentricity, or directrix
- Proofs
- A chain of reasoning
- Loci

Example 1: Knowledge of the geometry of conic sections

Any parabola can be defined as the locus of a point which moves so that it is equidistant from a fixed point (the focus), and a fixed line (the directrix).

A given parabola has its vertex at $V(3, 2)$ and its focus at $F(6, 2)$. Let $P(x, y)$ be a point on the parabola and the line L be the directrix.

Write expressions for the lengths PN and PF .

Use these to show that the equation of the parabola is $(y - 2)^2 = 12(x - 3)$.

The directrix is the same distance from the vertex as the focus point is. In this example, the directrix happens to be the y -axis.

The length of $PN = x$.

The length of $PF = \sqrt{(x - 6)^2 + (y - 2)^2}$.

Since $PN = PF$:

$$x = \sqrt{(x - 6)^2 + (y - 2)^2}$$

$$x^2 = (x - 6)^2 + (y - 2)^2$$

$$x^2 = x^2 - 12x + 36 + (y - 2)^2$$

$$x^2 - x^2 + 12x - 36 = (y - 2)^2$$

$$(y - 2)^2 = 12x - 36$$

$$(y - 2)^2 = 12(x - 3)$$

